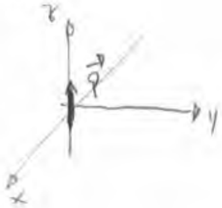
Dipolo



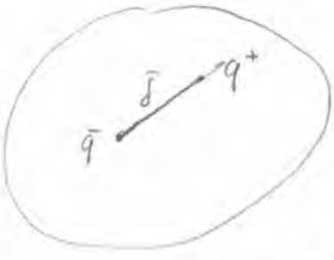
$\bar{p} = q\,\bar{\delta}$

$r \gg \delta$

$$V(r,\theta) = \frac{1}{4\pi\varepsilon_0}\,\frac{\bar{p}\cdot\bar{r}}{r^3}$$

$$\bar{E}(\bar{r}) = -\bar{\nabla} V = \frac{1}{4\pi\varepsilon_0}\left\{\frac{3(\bar{p}\cdot\bar{r})\,\bar{r}}{r^5} - \frac{\bar{p}}{r^3}\right\}$$

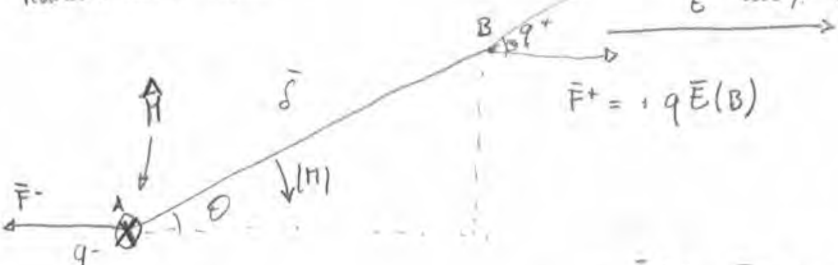
$\bar{F} = q\bar{E}$



Esiste un momento

MENCUCCINI

Momento di un dipolo



$\bar{F}^+ = +q\bar{E}(B)$

$\boxed{\bar{F} = \bar{F}_+ + \bar{F}_- = 0}$

Ma la coppia di Forze del braccio $\neq 0$

se $\bar{E}(A) = \bar{E}(B) \Rightarrow |\bar{F}^-| = |\bar{F}^+|$

$\bar{F}^- = -q\bar{E}(A)$

$\bar{M} = \bar{r}\times\bar{F}$

$\bar{M} = \vec{\delta}\times q\vec{E} = q\,\vec{\delta}\times\bar{E} = \vec{p}\times\bar{E}$

$\bar{M} = \bar{p}\times\bar{E} = -p\,E\,\mathrm{sen}(\theta)\,\hat{z}$

si prende $\hat{M}$ in A perché comunque il dipolo è molto piccolo quindi trascurabile in generale

oscillazioni

$I\ddot{\theta} = M = pE\,\mathrm{sen}(\theta) \sim pE\,\theta$

momento d'inerzia

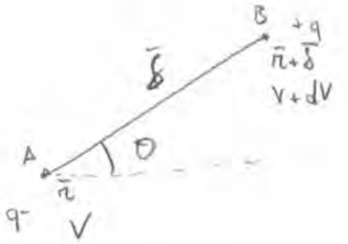
Eq dell'oscillatore armonico

$\omega_0 = \sqrt{\frac{pE}{I}} \Rightarrow T = \frac{2\pi}{\omega_0}$

Se $\underline{\bar{E}(A) \neq \bar{E}(B)} \Rightarrow \bar{F} \neq 0 \neq \bar{F}_+ + \bar{F}_-$

$\Rightarrow \bar{F} = q[\bar{E}(A) - \bar{E}(B)] \neq 0$

Energia Potenziale del dipolo nel campo $\bar{E}(\bar{r})$



$$U = qV \qquad U = U_- + U_+$$

$$U = -qV(\bar{r}) + qV(\bar{r}+\bar{\delta}) = q[V(\bar{r}+\bar{\delta}) - V(\bar{r})] = q\,dV =$$

$$U = q\,dV = q\underbrace{\bar{\nabla}V}_{=-\bar{E}}\cdot\bar{\delta} = -q\bar{E}\cdot\bar{\delta} = -q\bar{\delta}\cdot\bar{E} = -\bar{p}\cdot\bar{E} =$$

$U(x, y, z, \theta)$

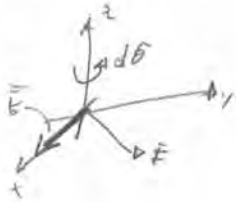
$$= -\bar{p}\,\underset{E(x,y,z)}{\underset{\uparrow}{\bar{E}}}\cos(\theta) \longrightarrow \text{MINIMIZZA L'ENERGIA QUINDI}$$

$\theta \longrightarrow 0$

ENERGIA MINIMA

Se trascuriamo l'energia Cinetica

il Lavoro $\longrightarrow$ $dL = \bar{F}\,d\bar{l} + \bar{M}\cdot d\bar{\theta} = -dU = -\frac{\partial U}{\partial l}dl - \frac{\partial U}{\partial \theta}d\theta$



MENOCCONI

$$\bar{F}\,d\bar{l} = -\frac{\partial U}{\partial l}dl$$

$$\bar{M}\cdot d\bar{\theta} = -\frac{\partial U}{\partial \theta}d\theta$$

$$\bar{F}\,d\bar{l} = -\left.\frac{\partial U}{\partial l}\right|_{\theta=cost} dl = -\frac{dU}{}_{\theta=cost} = -\left.\bar{\nabla}U\right|_{\theta=cost}\cdot d\bar{l}$$

$$\begin{cases} \bar{F} = -\bar{\nabla}U_{\theta=cost} = \bar{\nabla}(\bar{p}\cdot\bar{E}) \\ M = \bar{p}\cdot\bar{E} \end{cases}$$

$$\bar{F} = \bar{\nabla}(\bar{p}\cdot\bar{E}) = \bar{\nabla}(p_x E_x + p_y E_y + p_z E_z)$$

se il campo è conservativo $\bar{\nabla}\times\bar{E} = 0$

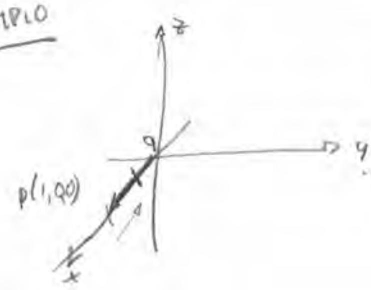
allora vale $\frac{\partial E_x}{\partial y} = \frac{\partial E_y}{\partial x}$

$$F_x = \frac{\partial}{\partial x}(p_x E_x + p_y E_y + p_z E_z) = p_x\frac{\partial E_x}{\partial x} + p_y\underbrace{\frac{\partial E_y}{\partial x}}_{=\frac{\partial E_x}{\partial y}} + p_z\underbrace{\frac{\partial E_z}{\partial x}}_{=\frac{\partial E_x}{\partial z}} =$$

Per il Th di Swartz

$$= p\bar{\nabla}E_x \quad \Rightarrow \quad F = \bar{\nabla}(\bar{p}\cdot\bar{E}) = (\bar{p}\cdot\bar{\nabla}E_x,\ \bar{p}\cdot\bar{\nabla}E_y,\ \bar{p}\,\bar{\nabla}E_z)$$

ESEMPIO



$q > 0$

$\bar{p} \quad \bar{F}, \bar{M}$

$$\bar{p} = \begin{cases} \text{A} & \bar{p} = p\,\hat{y} \\ \text{B} & \bar{p} = p\,\hat{x} \end{cases}$$

$$\begin{cases} \bar{F} = \bar{\nabla}(\bar{p}\cdot\bar{E}) \\ \bar{M} = \bar{p}\times\bar{E} \end{cases}$$

il calcolo va fatto con tutto
E non con le sue coordinate

$$\bar{E}(x,y,z) = \frac{q}{4\pi\varepsilon_0}\frac{\bar{r}}{r^3}$$

A $\quad \bar{F} = \bar{\nabla}\left(p\,\hat{y}\cdot\frac{q}{4\pi\varepsilon_0}\frac{\bar{r}}{r^3}\right) =$

$(0,p,0) \qquad \bar{r} = \bar{r}(x,y,z)$

$$= \bar{\nabla}\left(\frac{pq}{4\pi\varepsilon_0}\frac{y}{r^3}\right) = \frac{pq}{4\pi\varepsilon_0}\bar{\nabla}\left(\frac{y}{r^3}\right)$$

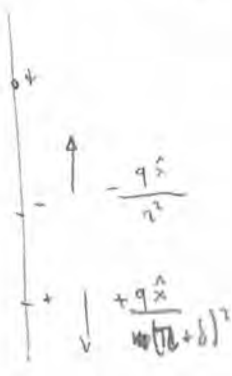
$$= \bar{\nabla}\left((0,p,0)\cdot\frac{q}{4\pi\varepsilon_0}\frac{(x,y,z)}{r^3}\right)$$

$$\bar{\nabla}\left(\frac{y}{r^3}\right) = \underbrace{\bar{\nabla}y}_{=\hat{y}}\,\frac{1}{r^3} + y\,\underbrace{\bar{\nabla}\left(\frac{1}{r^3}\right)}_{=-3\frac{\bar{r}}{r^5}}$$

$$\bar{F} = \frac{pq}{4\pi\varepsilon_0}\left\{\frac{\hat{y}}{r^3} - \frac{3y}{r^5}\bar{r}\right\}$$

B $\quad \bar{p} = p\,\hat{x}$

$$\bar{F} = \frac{qp}{4\pi\varepsilon_0}\bar{\nabla}\left(\frac{x}{r^3}\right) = \frac{qp}{4\pi\varepsilon_0}\left\{\frac{\hat{x}}{r^3} - \frac{3x\,\bar{r}}{r^5}\right\} = \frac{pq}{4\pi\varepsilon_0}\left\{\hat{x} - 3\hat{x}\right\} = -\frac{3pq}{4\pi\varepsilon_0}\hat{x}$$



A

$$\bar{M} = \bar{p}\times\bar{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & p & 0 \\ E_x & E_y & E_z \end{vmatrix} = \hat{x}\,pE_z + \hat{y}\,0 - \hat{z}\,p\,E_x$$

Sviluppo in serie di Multipoli

Data una qualunque distribuzione di cariche $V(r)$

$$V(r) = \frac{1}{4\pi\varepsilon_0} \sum_{1}^{N} {}_k \frac{\alpha_k}{(r)^k}$$

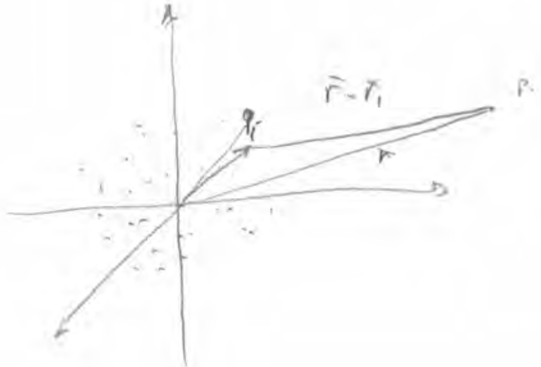
$\frac{1}{r}$ MONOPOLO

$\frac{1}{r^2}$ DIPOLO

$\frac{1}{r^3}$ QUADRUPOLO

DOMINANZA

stimabile con analisi sperimentali



$$|\bar{r} - \bar{r}_i| \sim \bar{r}$$

$$V(P) = \frac{1}{4\pi\varepsilon_0} \sum_{i}^{n} \frac{q_i}{|\bar{r} - \bar{r}_i|} = \frac{Q_{TOT}}{r}$$

se $Q_{TOT} = 0$ Non va bene questa sempl. BRUTALE

Th Carnot

$$\frac{1}{|\bar{r} - \bar{r}_i|} = \frac{1}{\sqrt{(\bar{r} - \bar{r}_i)(\bar{r} - \bar{r}_i)}} = \frac{1}{\sqrt{r^2 + r_i^2 - 2\bar{r}\cdot\bar{r}_i}} = \frac{1}{r} \frac{1}{\sqrt{1 - 2\frac{\bar{r}\cdot\bar{r}_i}{r^2} + \left(\frac{r_i}{r}\right)^2}} \simeq \frac{1}{r} \frac{1}{\sqrt{1 - 2\frac{\bar{r}\cdot\bar{r}_i}{r^2}}} \simeq$$

$$o(1) \quad o\left(\frac{r_i}{r}\right) \quad \cancel{o\left(\left(\frac{r_i}{r}\right)^2\right)}$$

$$\simeq \frac{1}{r}\left(1 + \frac{\bar{r}\cdot\bar{r}_i}{r^2}\right)$$

$$V(P) = \frac{1}{4\pi\varepsilon_0} \frac{1}{r} \sum_i q_i \left(1 + \frac{\bar{r}\cdot\bar{r}_i}{r^2}\right) = \frac{1}{4\pi\varepsilon_0} \frac{\sum_i q_i}{r} + \frac{1}{4\pi\varepsilon_0} \frac{\sum_i q_i \vec{r}_i \cdot \bar{r}}{r^3}$$

$\vec{p} = \sum_i q_i \vec{r}_i$ momento di dipolo

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{\vec{p}\cdot\vec{r}}{r^3} + \frac{Q_{TOT}}{r}\right)$$

CENTRO DI CARICA

$$\begin{cases} Q = \sum_i q_i \longrightarrow \int \rho \, d\tau \\ \vec{p} = \sum_i q_i \bar{r} \longrightarrow \int \rho(\bar{r}) \bar{r} \, d\tau \end{cases}$$

(il parallelo con meccanica è forte)